

# LOGIQUE

## Dédution naturelle-NK

### 1-Axiomes

$$\frac{}{\Gamma, A \vdash A} \text{ax}$$

### 2-Règles structurelles

$$\frac{\Gamma \vdash A}{\Gamma, B \vdash A} \text{aff}$$

### 3-Règles des connecteurs

$$\frac{\Gamma \vdash A \wedge B}{\Gamma \vdash A} \wedge_{\text{elim}}^{\ell}$$

$$\frac{\Gamma \vdash A \wedge B}{\Gamma \vdash B} \wedge_{\text{elim}}^r$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \wedge B} \wedge_{\text{intro}}$$

$$\frac{\Gamma \vdash A \vee B \quad \Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma \vdash C} \vee_{\text{elim}}$$

$$\frac{\Gamma \vdash A}{\Gamma \vdash A \vee B} \vee_{\text{intro}}^{\ell}$$

$$\frac{\Gamma \vdash B}{\Gamma \vdash A \vee B} \vee_{\text{intro}}^r$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash A \rightarrow B}{\Gamma \vdash B} \rightarrow_{\text{elim}}$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \rightarrow B} \rightarrow_{\text{intro}}$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash \neg A}{\Gamma \vdash \perp} \neg_{\text{elim}}$$

$$\frac{\Gamma, A \vdash \perp}{\Gamma \vdash \neg A} \neg_{\text{intro}}$$

$$\frac{\Gamma, \neg A \vdash \perp}{\Gamma \vdash A} \perp_{\text{classic}}$$

### 4-Règles des quantificateurs

$$\frac{\Gamma \vdash \forall x A}{\Gamma \vdash A[x:=t]} \forall_{\text{elim}}$$

$$\frac{\Gamma \vdash A}{\Gamma \vdash \forall x A} \forall_{\text{intro}} \text{ (if } x \notin \text{FV}(\Gamma)\text{)}$$

$$\frac{\Gamma \vdash \exists x A \quad \Gamma, A \vdash B}{\Gamma \vdash B} \exists_{\text{elim}} \text{ (if } x \notin \text{FV}(\Gamma, B)\text{)}$$

$$\frac{\Gamma \vdash A[x:=t]}{\Gamma \vdash \exists x A} \exists_{\text{intro}}$$

## Dédution naturelle intuitionniste NJ

### 1-Axiomes

$$\frac{}{\Gamma, A \vdash A} \mathbf{ax}$$

### 2-Règles structurelles

$$\frac{\Gamma \vdash A}{\Gamma, B \vdash A} \mathbf{aff}$$

### 3-Règles des connecteurs

$$\frac{\Gamma \vdash A \wedge B}{\Gamma \vdash A} \wedge_{\text{elim}}^{\ell} \qquad \frac{\Gamma \vdash A \wedge B}{\Gamma \vdash B} \wedge_{\text{elim}}^r \qquad \frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \wedge B} \wedge_{\text{intro}}$$

$$\frac{\Gamma \vdash A \vee B \quad \Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma \vdash C} \vee_{\text{elim}} \qquad \frac{\Gamma \vdash A}{\Gamma \vdash A \vee B} \vee_{\text{intro}}^{\ell} \qquad \frac{\Gamma \vdash B}{\Gamma \vdash A \vee B} \vee_{\text{intro}}^r$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash A \rightarrow B}{\Gamma \vdash B} \rightarrow_{\text{elim}} \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \rightarrow B} \rightarrow_{\text{intro}}$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash \neg A}{\Gamma \vdash \perp} \neg_{\text{elim}} \qquad \frac{\Gamma, A \vdash \perp}{\Gamma \vdash \neg A} \neg_{\text{intro}}$$

$$\frac{\Gamma \vdash \perp}{\Gamma \vdash A} \perp_{\text{elim}}$$

### 4-Règles des quantificateurs

$$\frac{\Gamma \vdash \forall x A}{\Gamma \vdash A[x:=t]} \forall_{\text{elim}} \qquad \frac{\Gamma \vdash A}{\Gamma \vdash \forall x A} \forall_{\text{intro}} \text{ (if } x \notin \text{FV}(\Gamma))$$

$$\frac{\Gamma \vdash \exists x A \quad \Gamma, A \vdash B}{\Gamma \vdash B} \exists_{\text{elim}} \text{ (if } x \notin \text{FV}(\Gamma, B)) \qquad \frac{\Gamma \vdash A[x:=t]}{\Gamma \vdash \exists x A} \exists_{\text{intro}}$$